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A CAMERA FOR STUDYING PROJECTILES IN FLIGHT

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A CAMERA FOR STUDYING PROJECTILES IN FLIGHT.

By H. L. Curtis, W. H. Wadleigh, and A. H. Sellman.

ABSTRACT.

In order to obtain unblurred pictures of a rapidly moving object like a projectile in flight, it is necessary to use a camera in which the photographic film moves with approximately the same velocity as the image of the object. A series of pictures can be obtained by using several lenses of the same kind, mounting them in a line which is perpendicular to the direction of motion of the film, and arranging a focal plane shutter so as to make successive exposures by the different lenses.

The camera described in this article will not only take pictures of objects moving with high speed, but will also determine their velocity. The number of pictures will depend on the number of lenses, 50 pictures per second per lens being easily obtained. If the object is moving uniformly, the error in the measured velocity will not be as much as 1 per cent.

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I. INTRODUCTION.

It is possible with commercial cameras to obtain the picture of a projectile in flight provided an exposure can be made at the right instant. However, even with the shortest possible exposure, the image is somewhat blurred on account of the rapid motion of the projectile. If a commercial moving-picture camera is used, it is probable that one and only one blurred picture will be obtained.

Special cameras have been developed which will photograph moving projectiles, but each of these was designed for a specific and very limited use. These can be divided into two classes—those which use an electric spark for illumination and those which depend on daylight. A complete bibliography of articles on

photographing projectiles by an electric spark is given in an article by Crantz and Glatzel.[1] A camera using daylight is described by Cles and Swoboda.[2]

The camera described in this article was designed to take several pictures of the projectile and to have these pictures as clear-cut as possible. In addition, the camera is so designed that the time between pictures can be accurately measured, so that the velocity of the projectile can be determined. With this camera it should be possible to study the yaw of the projectile, its velocity, and its speed of rotation. It has also been used to study the blast; that is, the hot gases which issue from the gun when a projectile is fired.

II. DESCRIPTION OF CAMERA.

There are three distinct objects to be accomplished, viz, (1) the photographing of a clear image of the projectile, (2) the obtaining of a number of pictures, and (3) the determining of the time between pictures. The method of accomplishing each of these objects will be described independently.

1. METHOD OF OBTAINING A CLEAR IMAGE.

The method of obtaining a clear picture is shown in Figure 1. The sensitive film is mounted on a drum which can be made to rotate with such a speed that the film moves with the estimated speed of the image of the projectile. The focal plane shutter drum rotates in the opposite direction from the film drum and at a speed several times as great. Both of these drums run continuously and uniformly. An electrical circuit is arranged to open the main shutter by means of the attached magnet just before the projectile comes into the field of the camera. An attachment on the film drum breaks the electrical circuit, so as to close the shutter when the film drum has made one revolution.

When the projectile comes into the field of the camera with the main shutter open, an image is formed on the focal plane shutter drum. The speed of this shutter drum is such that before the projectile leaves the field of the camera at least one of the openings in the shutter drum will pass the image, thus exposing the film. As the film and the projectile image are moving approximately with the same speed and in the same direction, there is no blurring or distortion of the projectile picture.

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[1] Die Verwendung von Gleichstrom-Löschfunkenstrecken zur kinematographischen Aufnahme ballistischer und physikalischer Vorgänge. Verh. der Deut. Phys. Gesellschaft., **14**, p. 525; 1912. This method has never been used with large projectiles.

[2] Kinematographische Aufnahme von Geschützprojektilen während der Bewegung bei Tageslicht. Akad. Wein. Math. Nat. Klasse, Sitzungsberichte, 123, IIA, p. 757; 1914. A translation of this article is published in the Journal of U. S. Artillery, 48, p. 91; 1917.

The pictures of stationary objects will be somewhat blurred and distorted, since the film is moving continuously. By making the openings in the shutter drum very narrow and its speed high the blurring can be reduced at the expense of the intensity of the picture. The distortion is such as to increase the size of an object in the direction of motion of the film. It can be reduced by increasing the speed of the shutter drum, but this decreases the time of exposure and hence the intensity of the picture.

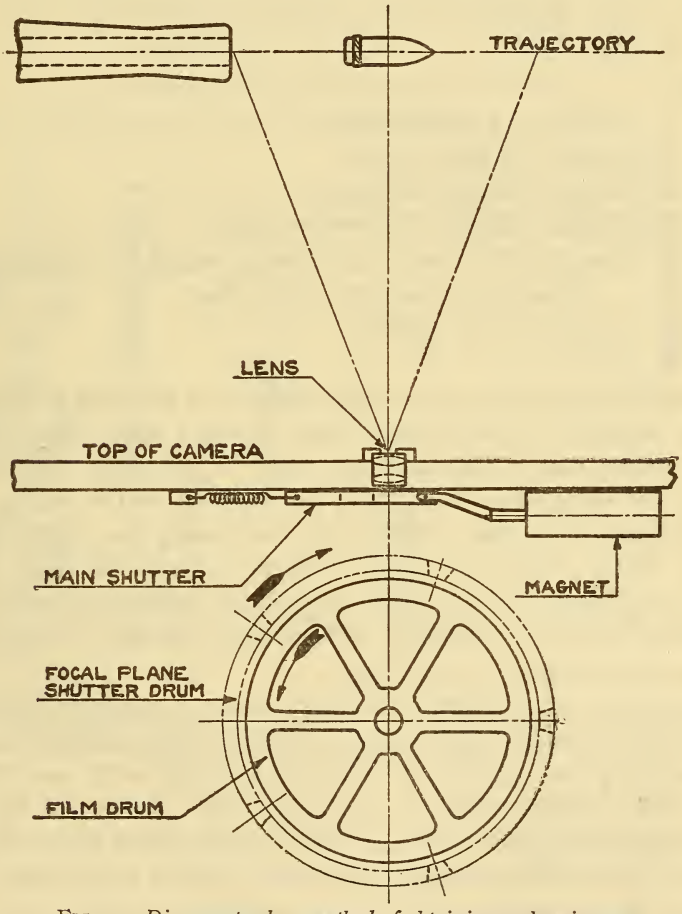

FIG. 1.—*Diagram to show method of obtaining a clear image.*
When the film moves with the same velocity as the image of the projectile, there is no blurring of the picture.

In order to give a mathematical expression for the blurring and distortion, it is necessary to give definitions to these quantities. The blurring is defined as the movement relative to the film of a point in the image during the time that this point is exposed. The distortion is the ratio of the difference between the length of the image of an object and that of its picture to the length of the image.

Let

b = blurring

and

d = distortion

Then

$$b = s\left(\frac{V_f - V_i}{V_s}\right)$$

and

$$d = \frac{\delta l}{l} = \frac{V_f - V_i}{V_s}$$

Where

s = width of a shutter slot
V_f = linear velocity of film
V_i = linear velocity of image
V_s = linear velocity of shutter drum
l = length of a line in the image which is parallel to the direction of motion of the drum
l' = length of l in picture
$\delta l = l' - l$

It will be noted that, when the velocity of the film is the same as the velocity of the image, both blurring and distortion are zero. When this is not the case, both the blurring and the distortion are reduced by increasing the velocity of the shutter, and the blurring is further reduced by decreasing the width of the shutter slots. However, both of these decrease the time of exposure, so that it is impossible to entirely eliminate blurring and distortion when the velocity of the film can not be made the same as the velocity of the image.

It follows from the above that the time of exposure of the image of the projectile can be made relatively long without blurring or distorting the picture by making the slots wide and the velocity of the shutter drum low, provided the velocities of the films and image are equal. In fact, if the speed of the film is exactly equal to that of the image, no shutter drum is necessary. In practice these conditions can not be realized; also, for many purposes, it is necessary to have on the same film pictures of stationary and moving objects. Hence the shutter drum is essential, and the best results will be obtained by making the speed of this drum relatively high and the slots narrow, while still permitting enough light to reach the film to give sufficient density in the picture.

In the above discussion it is assumed that there are several slots in the shutter drum, but the distance between them has not been considered. With the proper distance between the slots, there will be no overlapping of pictures and all the film will be used. The equation which gives this distance is—

$$D + s = L\left(1 + \frac{V_s}{V_f}\right) \qquad (1)$$

where

D = distance between centers of slots,
s = width of a slot,
L = length across light cone at the film,

and the velocities are those indicated above.

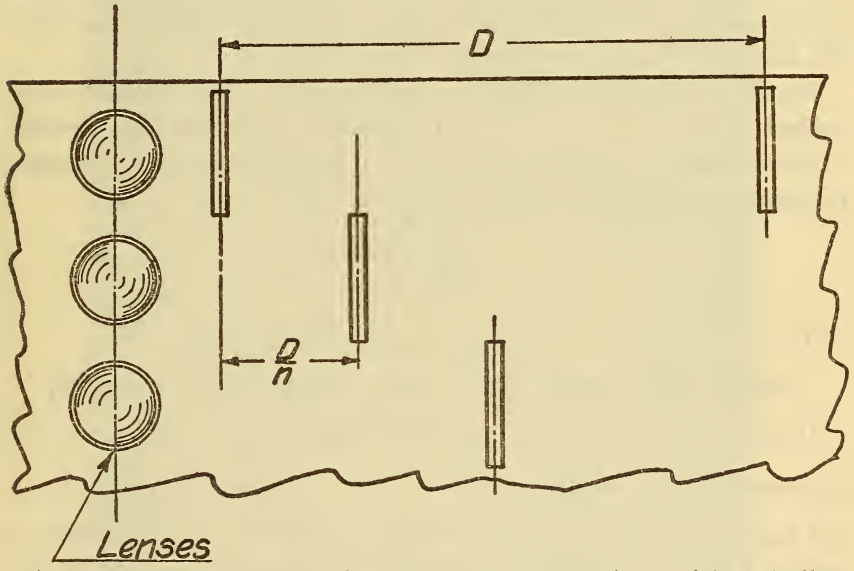

FIG. 2.—*Diagram to show method of obtaining a number of pictures of the projectile.* By increasing the number of lenses and properly spacing the slots in the focal plane shutter, several thousand pictures per second can be taken.

It will be noted that the proper distance between the slots depends on the dimensional constants of the instrument and on the relative velocities of the two rotating drums. Hence, if the two drums are geared together or driven by the same motor, the pictures will always be in the same relative succession regardless of the absolute velocity of the film.

2. METHOD OF OBTAINING A NUMBER OF PICTURES.

The method of obtaining a number of pictures is shown in Figure 2. A number of similar lenses are mounted in a line perpendicular to the direction of motion of the film. The film is of sufficient width so that the pictures from all the lenses are formed

on the same film. The slots in the shutter drum are so placed that the time between pictures is the same. That is, if there are n lenses and if the linear distance between two slots which pass under the same lens is D, then the distance between two slots which pass successively under two adjacent lenses is D/n.

The pictures of a stationary object will lie in a diagonal across the film, but repeated for each lens as long as the main shutter is open. If the image of the projectile has the same velocity as the film, then each lens will take only one picture of the projectile, and the pictures from the different lenses will lie across the film in a line perpendicular to the direction of motion. In Figure 3 there is reproduced a photograph in which the pictures of the projectile are very nearly in a line across the film. This shows that the film and image had nearly the same velocity.

By this arrangement of lenses any desired number of pictures per second can be taken. If N is the number of pictures per second, n the number of lenses, and P the distance between successive pictures of a stationary object by one lens, then

$$N = \frac{nV_f}{P}$$

With five lenses 250 pictures per second have been taken.

3. METHOD OF DETERMINING THE TIME BETWEEN PICTURES.

It has just been shown that the time between pictures of a stationary object is $\frac{P}{nV_f}$. Hence, to get this time it is necessary to get the velocity of the film and the distance in the direction of motion between successive pictures of a stationary object. If the blurring is small, the distance P can be measured on a comparator. The velocity of the film can be determined by photographing on the film flashes of light from a tuning fork. For this purpose the film drum extends a short distance (about 1 cm) beyond the shutter drum. It is on this portion of the film that the tuning-fork record is made.

The principle of the method [3] is shown in Figure 4. A slotted vane is attached to each of the prongs of a tuning fork, the slots being so adjusted that they coincide when the fork is at rest. This slot is brilliantly illuminated by any suitable source of light. An optical system is arranged to throw an image of the slot on

[3] This method is described and the sources of error discussed in a paper by Harvey L. Curtis and Robert C. Duncan, B. S. Sci. Papers, 19, p. 17 (S. P. 470).

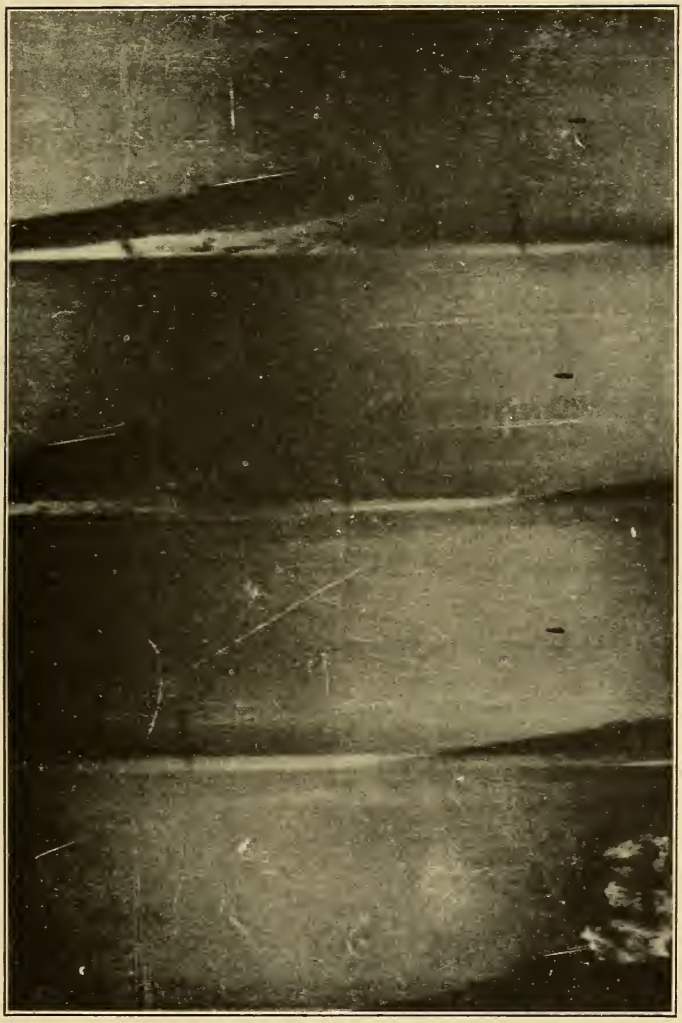

FIG. 3.—*Enlarged photograph of projectile in flight.*

In this picture the velocity of the film is nearly the same as the velocity of the image. Note the distinctness in the projectile pictures in comparison with the telegraph pole.

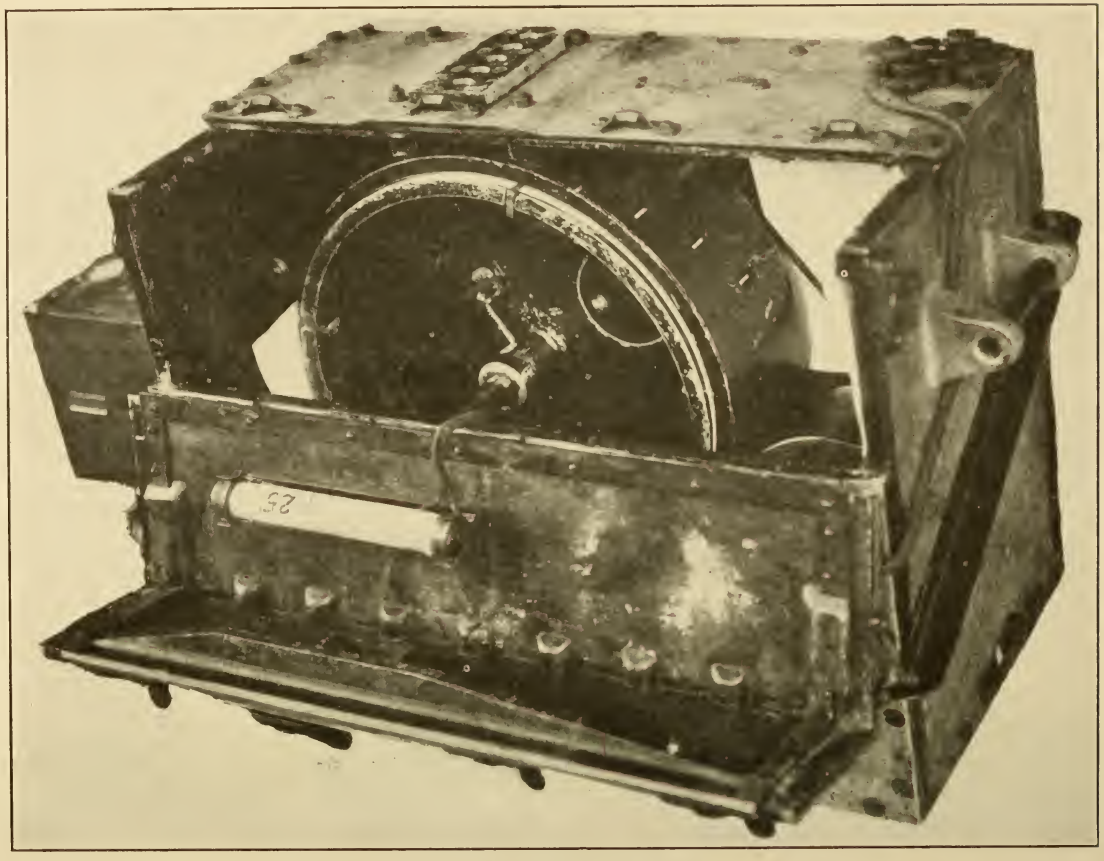

FIG. 6.—*Photograph of the camera.*

the photographic film. When the fork is vibrating, flashes of light will be thrown on the film when the two slots coincide. If the film is moving, the flashes will produce a time scale on the film. By measuring the distance between the timing lines and knowing the rate of the fork the velocity of the film can be determined.

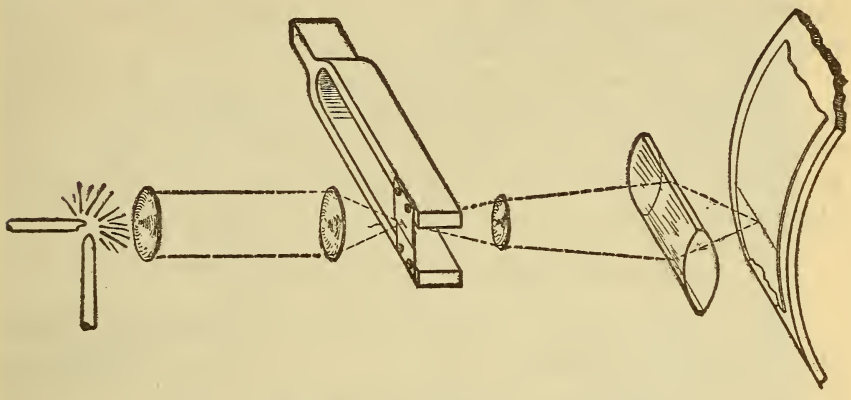

FIG. 4.—*Diagram of tuning fork and optical system.*
The opening produced by the two coincident slots in the vanes of the fork is brilliantly illuminated. A long, narrow image of this opening is thrown on the photographic film by means of one spherical and one cylindrical lens. With the fork vibrating and the film moving, lines will be produced on the film with a definite time interval between them.

A method of attaching the tuning fork and auxiliary apparatus to the camera is shown in Figure 5. The tuning fork is placed

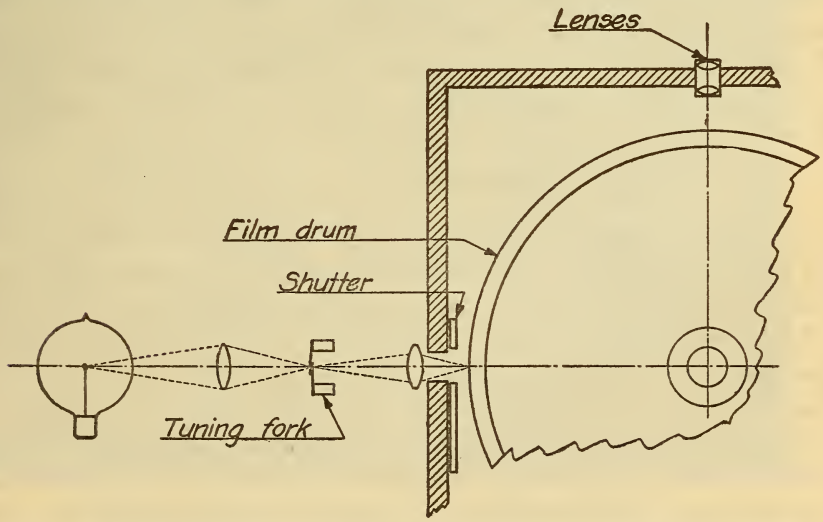

FIG. 5.—*Diagram showing the tuning fork and optical system attached to the camera.*
For mechanical reasons, this is attached 90° from the lenses.

at 90° from the lenses. The illumination is by an incandescent lamp, which can be overvolted during the time that the exposure is made. A self-driven fork is essential. We have satisfactorily used a 250-cycle fork for this purpose.

The three features have been described separately, but the entire instrument is assembled in a single case. This case also contains the motor for driving the drums. The shaft for the film drum must be more than twice as long as the width of the drum, so that this drum can be moved from under the shutter drum for putting on and taking off the films. A picture of the assembled instrument is shown in Figure 6.

Wide-angle lenses should be used when it is desired to photograph a considerable length of the trajectory of a projectile; also, the lenses must be as nearly as possible of the same focal length in order to give the same magnification in all of the pictures. Means must also be provided for focusing the lenses.

III. APPLICATIONS OF THE CAMERA.

This camera can be used for studying any moving object. However, its principal field is where the object is moving in a given direction with a speed which is approximately known. Up to the present it has been used chiefly to study phenomena associated with the flight of projectiles.

The value of the results which can be obtained in any particular case can only be determined by a careful analysis. In some cases more satisfactory results were obtained than was anticipated, while in other cases where little difficulty was expected the results have been unsatisfactory. Hence, the important applications which have been suggested will be discussed and the experimental results given, if any have been obtained.

1. PHOTOGRAPHING THE BLAST.

Questions frequently arise which involve the relative velocity of the projectile and blast. Pictures taken as the projectile is leaving the gun will frequently answer these questions. The shape of the blast is well shown in Figures 7 and 8. These were taken of the same gun on the same day and show the differences that must be expected in pictures of this kind. By chance, in Figure 7, picture No. 1 was taken just as the projectile was emerging from the muzzle. In the original the gas which preceded the projectile can be seen. In picture No. 3 the projectile has left the blast. In the succeeding pictures the projectile has a much higher velocity than the blast.

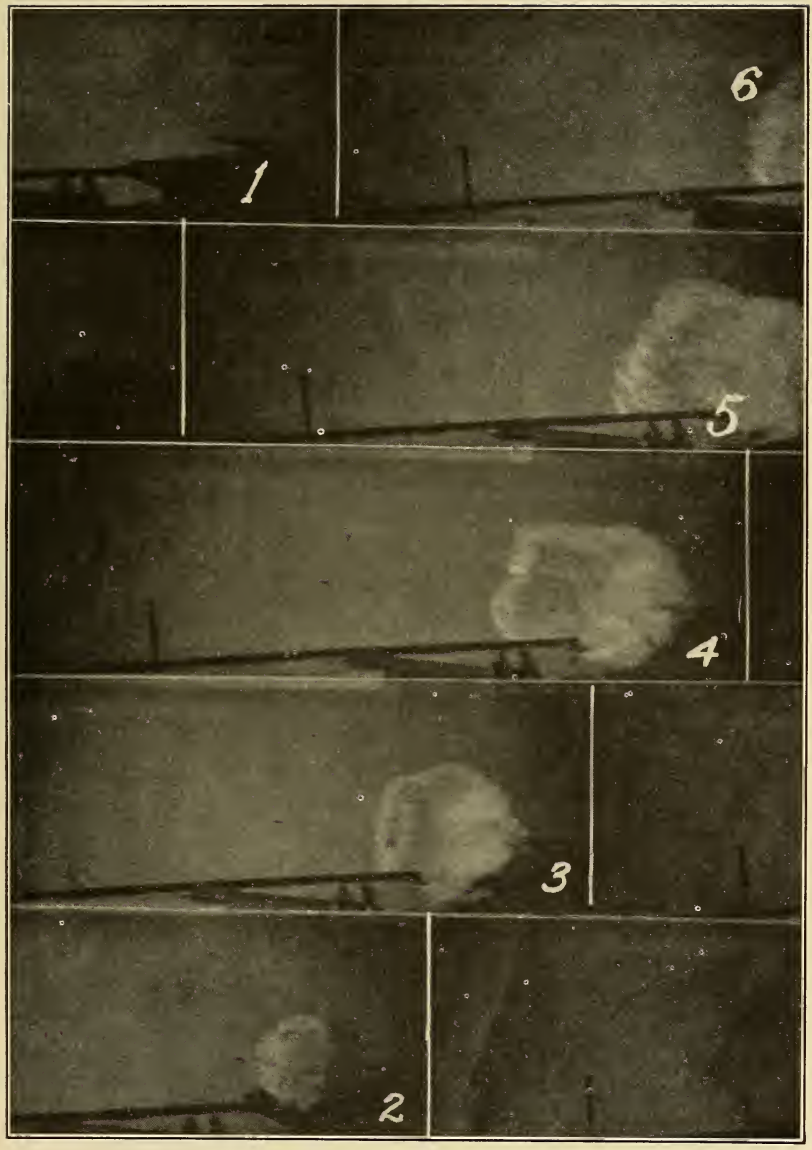

FIG. 7.—*Enlarged photograph showing projectile and blast.*

In picture 1 the projectile can be seen just emerging from the gun. In picture 3 it has completely left the blast. In the following pictures of the projectile, the relatively slow movement of the blast is seen.

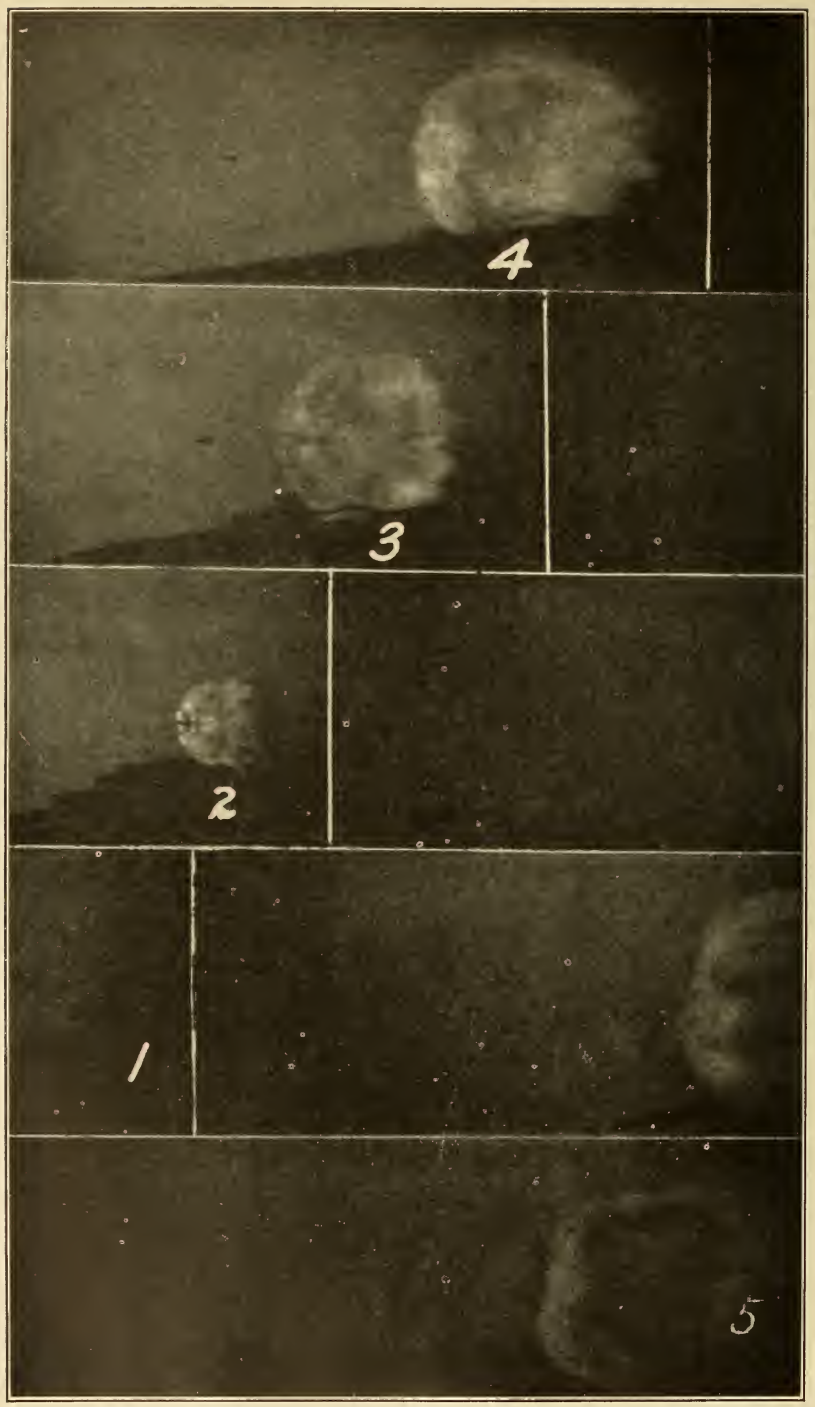

FIG. 8.—*Enlarged picture showing projectile and blast.*

In picture 1 the projectile has not reached the muzzle of the gun. In picture 2 the projectile is several feet in front of the gun and is just emerging from the blast.

In Figure 8 picture No. 2 shows the projectile just emerging from the blast. When the projectile leaves the gun, the powder gases at first have a higher velocity than the projectile, so that when the projectile is a short distance from the muzzle it is completely surrounded by the blast. However, the deceleration of the gases is very rapid, so that in a few feet the projectile leaves the gases. The distance that the projectile is from the gun when it leaves the blast can be determined if one is fortunate enough to obtain a picture so near the time of emergence as is the case in picture No. 2 of Figure 8.

2. PHOTOGRAPHING A PROJECTILE PENETRATING ARMOR.

If a large projectile is fired into good armor with a velocity of 1,000 ft./sec. it will penetrate approximately 1 foot. Assuming uniform deceleration, two thousandths of a second would elapse from the time the projectile struck the armor till it is completely stopped. In this interval several pictures should be taken in order to obtain satisfactory information concerning the behavior of the projectile. However, the field does not need to be large. Hence, by designing a camera in which the field is 1 cm long on the film, the velocity of the film, V_f, is 250 cm/sec., and the number of lenses, n, is 10, the number of pictures per second N is

$$N = \frac{nV_f}{L} = \frac{10 \times 250}{1} = 2{,}500$$

Hence, in two-thousandths of a second five pictures would be taken. This would be sufficient to give some idea of the behavior of the projectile.

The above assumes that proper lighting conditions could be obtained. The best pictures have been obtained with the projectile as a shadow against the sky as a background. It might be difficult to make this kind of an arrangement; also, at the point of impact, the armor is heated to incandescence. It will probably be necessary to so place the camera that very little of this light will enter the camera.

It has been suggested that the behavior of armor might be studied by firing the projectile with such velocity that it would completely penetrate the armor and then measure the decrease in velocity. This camera is well adapted for measuring the veloc-

ity after the projectile has passed through the armor, provided proper lighting conditions can be obtained. By using two cameras at right angles to each other, as shown in Figure 9, both the velocity and direction of motion can be obtained.

3. DETERMINING THE YAW OF A PROJECTILE.

The yaw of a projectile at any point is the angle between the axis of the projectile and the tangent to the trajectory at that point. It has often been suggested that it should be possible to

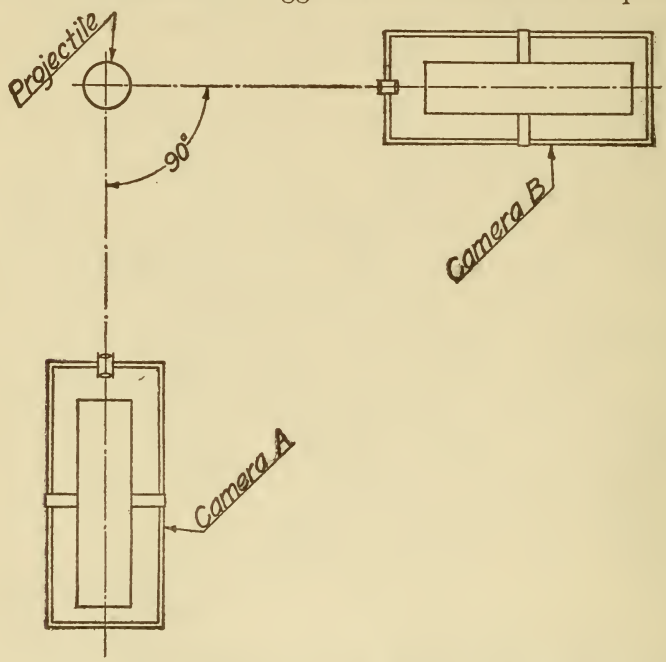

FIG. 9.—*Diagram to show use of two cameras to obtain both the velocity and direction of motion of a projectile.*

By placing the cameras as shown, both cameras will have the sky as a background.

measure the yaw by means of this camera, but a number of attempts have failed to give results. The reasons will be discussed below.

The period of precession of a large projectile is such that it is necessary to take pictures during approximately 50 feet of the trajectory to obtain satisfactory values of the yaw. With a lens having a 45° angular opening and a 2-inch focal length this will require a reduction of 1/360 in the size of the picture of the projectile. Hence, if the straight side of the projectile is 3 feet long, in the picture it is only 0.1 inch long. To detect a yaw of 1° it is necessary to measure a displacement of less than 0.002 inch of one end of this line. This might be done under very favorable conditions, but up to the present we have not been able to do so.

4. MEASUREMENT OF THE VELOCITY OF A PROJECTILE.

The velocity of any object, V_o, has the same ratio to the velocity of the image, V_i, as the length of the object, l_o has to the length of the image, l_i; that is,

$$V_o = V_i \frac{l_o}{l_i} = V_i m \tag{2}$$

where m is the magnification ratio of a line. If the film has exactly the same velocity as the image, then the length of the image equals the length of the picture, so that the formula becomes

$$V_o = V_f \frac{l_o}{l_p} = V_f m \tag{3}$$

where l_p is the length of the picture of the object and V_f is the velocity of the film. The method for measuring the velocity of the film has already been described, and the length of the picture can be determined by measurements on a comparator.

There are two reasons why it is impossible to obtain an accurate determination of the velocity of the projectile by the above formula. In the first place, it is very difficult, if not impossible, to adjust the velocity of the film to exactly that of the velocity of the image. In the second place, the projectile or other moving object is generally so small that it is very difficult to obtain a satisfactory value of the magnification ratio by making measurements of this object. It is shown below how these difficulties can be overcome.

Since the pictures are not distorted in a direction perpendicular to the motion of the film, the magnification can be determined by making measurements on a large object and its picture, which has been so placed that the axis in the picture is perpendicular to the direction of the motion of the film. In this way it is possible to use a large object, so that the measurements can be made with sufficient accuracy. This overcomes the second difficulty noted above.

If the velocity of the image is different from the velocity of the film, then

$$V_i = V_f \left[1 - \frac{L_1 (D+L)}{L (D+L_1)}\right] \tag{4}$$

Where

L_1 = distance in the direction of motion of the film between any two pictures of the moving object.

L = distance in the direction of motion of the film between the two pictures of a stationary object which were taken with the same lenses as were used in obtaining L_1.

D = circumferential distance between the slots which exposed these pictures.

Substituting these values in (2)

$$V_o = m\ V_f \left[1 - \frac{L_1(D+L)}{L(D+L_1)} \right] \tag{5}$$

This is a satisfactory formula for obtaining the velocity of the moving object. The accuracy which can be obtained by its use will be discussed later.

It is sometimes necessary to obtain the magnification by measurements of objects and their pictures whose axes are in the direction of motion of the film. In that case

$$m = 1 - \frac{L_1}{D+L_1}$$

In case the object on which these measurements are being made is at rest

$$L_1 = L$$

This formula for magnification, therefore, holds for any object whose axis is in the direction of motion of the film, regardless of whether it is moving or standing still.

The accuracy which can be obtained in measuring the velocity of a projectile depends, primarily, upon the accuracy of determining m and V_f. Both of these measurements are affected by the shrinkage of the film during the developing and drying process. The magnitude of the error which is likely to occur can be estimated by considering the separate elements which are measured.

$$mV_f = \frac{l_o}{l_p} \cdot \frac{s}{t} = \frac{l_o}{t} \cdot \frac{s}{l_p} \tag{6}$$

where s is the distance between timing lines and the other symbols have the same meaning as above.

The measurement of l_o and t do not in any way depend upon the film, but s and l_p are both made on the film. Hence, if the film shrinks uniformly, no error is introduced by this shrinkage. However, as s is measured along the film while l_p may be measured across the film, there is a chance that some error will be intro-

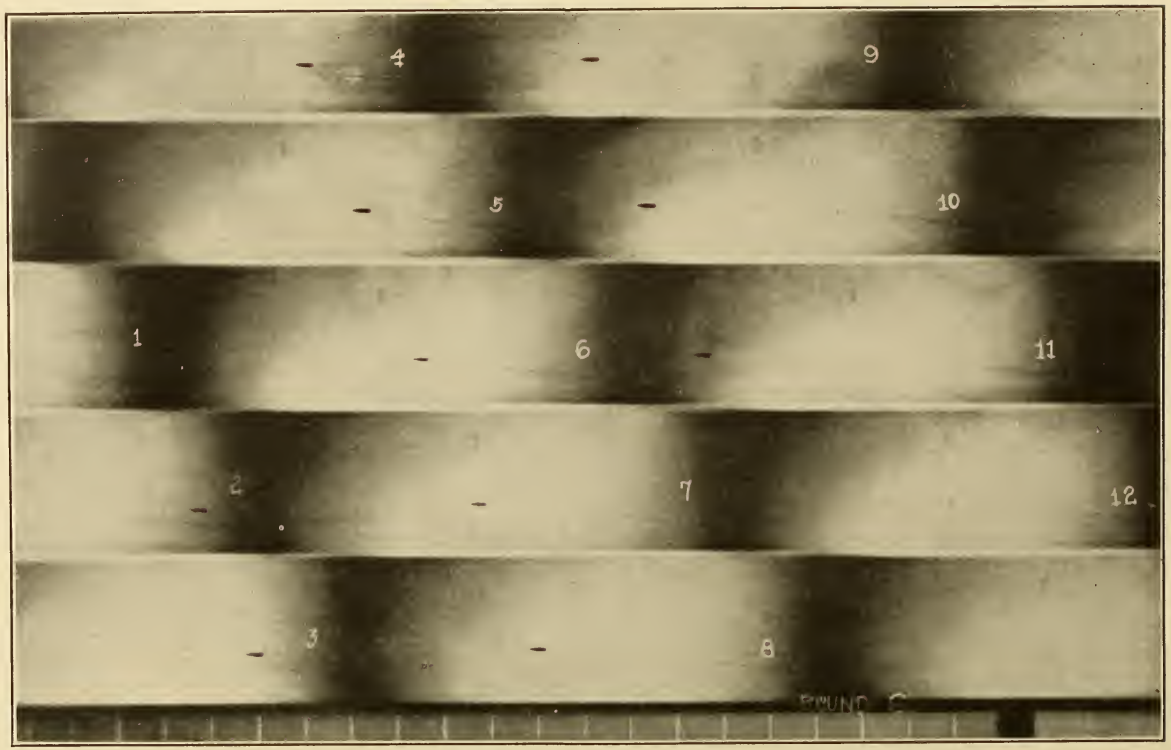

FIG. 10.—*One of the pictures used in determining the velocities given in Table 1.*

The magnification was determined from the known distance between the screens. In picture 7 the projectile is seen passing through one of the screens.

duced on account of unequal shrinkage. By making several measurements and averaging them the probability of such an error is reduced.

Of the four quantities given in equation (6) all can be measured with an accuracy of a tenth of a per cent with the possible exception of l_p. The length of l_p will probably be about 10 mm. Experienced observers can measure on a film with an accuracy of about 0.01 mm, which would give an accuracy of 0.1 per cent to l_p. However, conditions will frequently arise when it will be impossible to obtain this accuracy.

The correction term in equation (5) is zero when the velocity of the film is the same as the velocity of the image. When it is not zero, there is an error caused by the shrinkage of the film. By performing the indicated subtraction and factoring both the numerator and denominator the correction term may be written

$$\frac{D}{D+L_1} \cdot \frac{L-L'}{L}$$

A uniform shrinkage of the film will not change the value of the second of these two fractions but will change the first. If L_1 is one-tenth of D, then a shrinkage of 1 per cent in the film will introduce an error of 0.1 per cent in the result. This can be reduced by making the speed of the film approximately equal to the speed of the image. In any case this error will probably be less than the error caused by the unequal shrinkage of the film which was noted above.

The camera has been used several times to measure the velocity of projectiles. In Figure 10 is the reproduction of a film which was taken for the purpose of obtaining the velocity. The results of several measurements are given in Table 1. It is believed that the results are nearly, if not quite, as accurate as those obtained by the Boulenge chronograph.

TABLE 1.—Comparison of velocities.

Velocity by camera.	Velocity by Boulenge chronograph.
Ft./sec.	Ft./sec.
2,023	2,008
2,007	2,008
2,030	2,007

5. MEASUREMENT OF THE ROTATION OF A PROJECTILE.

By painting one or more white stripes the full length of the projectile it seems possible to obtain from a series of pictures taken at known intervals the speed of rotation. However, the exposures which have been used are too short to obtain anything but a shadow of the projectile against the sky as a background. By increasing the width of the slots a longer exposure can be made which would give more detail. There is a limit to this, however, as the motion in the direction of rotation is not compensated, and too long an exposure will only produce blurring. Hence, while the measurement of the rotation seems possible, a large amount of experimental work will be necessary to perfect the method.

IV. SUMMARY.

A camera suitable for obtaining a number of pictures of a rapidly moving object is described. The number of pictures per second which can be taken is very large, depending on the number of lenses which are used in the construction of the camera. In addition, the time between pictures can be accurately determined. By means of this camera it is possible not only to take a number of pictures of a rapidly moving object, like a projectile, but also to determine its velocity.

WASHINGTON, July 17, 1923.